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UNSTEADY FLOW OF GAS IN LAVAL NOZZLES WITH LOCAL SUPERSONIC ZONES

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Exact particular solutions of nonlinear equations defining unsteady transonic flows of gas are derived. These solutions are used for analyzing unstable flows in Laval nozzles with local supersonic zones.

Local supersonic zones in stationary flows were investigated with the use of the simplified equations for transonic flows of gas in [1] in plane nozzles and in [2] in nozzles of circular cross section. These solutions were then extended in [3] to three-dimensional flows in Laval nozzles. Since the investigation of variation of local supersonic zones with time is complicated by the nonlinearity of the equation for unstable transonic flows, hence even particular examples are of interest. A solution of this equation for an unsteady flow of the Taylor kind in a nozzle with two planes of symmetry was indicated in [3]. Similar solutions were later considered in [4] for unstable flows in plane nozzles. It was established that all of the above solutions can be extended. One of such generalized solutions of the transonic equation defining an unsteady flow of the Taylor kind in plane and axisymmetric Laval nozzles is presented in [5]. Here this solution is used for analyzing the variation of local supersonic zones with time (their onset, development, and joining at the nozzle axis, or the inverse process) for two classes of self-similar solutions.

1. 1) "Slow" unsteady transonic flows of perfect gas are defined by the system of equations

$$2u_\tau + uu_x - v_y - w_z = 0, \quad u_y = v_x, u_z = w_x, \quad v_z = w_y \tag{1.1}$$

where u, v and w are projections of the velocity vector on the axes of a Cartesian system of coordinates x, y, z, and τ is the time. The velocity potential is defined by the equation

$$2\varphi_{x\tau} + \varphi_x\varphi_{xx} - \varphi_{yy} - \varphi_{zz} = 0 \tag{1.2}$$

Differentiation of Eq. (1.2) with respect to x yields the equation $u = \varphi_x$. The solution of this equation which defines a flow with local supersonic zones in a Laval nozzle (for simplicity a nozzle with two mutually perpendicular planes of symmetry are considered here) is of the form

$$u = U(\xi, \tau) + a_1(\tau) y^2 + a_2(\tau) z^2, \quad x = m(\tau)\xi + n(\tau) + c_1(\tau) y^2 + c_2(\tau) z^2 \tag{1.3}$$

The equation for $U(\xi, \tau)$ is readily derived [5]. Since our aim is the investigation of variation of local supersonic zones (LSZ) with time, hence for simplicity only plane and axisymmetric flows will be considered. Extension of results obtained below to the three-dimensional case (1.3) is not difficult.

2) The system of Eqs. (1.1) admits a solution for the form

$$u = \tau^{n-1} u_* (x_*, y_*, t) + 2\lambda' (\tau), \quad v = \tau^{3/2(n-1)} v_* (x_*, y_*, t) + \frac{4\lambda'' (\tau)}{\omega + 1} y \qquad (1.4)$$

$$x_* = \frac{x - \lambda (\tau)}{\tau^n}, \quad y_* = y \tau^{-1/2(n+1)}, \quad t = \ln \tau$$

For u_* and v_* we seek solutions of the form (1.3)

$$u_* = mU (\xi, t) + 2c (2c - 1) y_*^2, \quad x_* = m\xi + cy_*^2 \qquad (1.5)$$

$$v_* = 2cm [2 (2c - 1) \xi - U (\xi, t)] y_* + \frac{8c (c - 1) (2c - 1)}{\omega + 3} y_*^3$$

where m, c and n are arbitrary constants, $\lambda (\tau)$ is an arbitrary function, and $\omega = 0$ for plane and $\omega = 1$ for axisymmetric flows. For function $U (\xi, t)$ we obtain the equation

$$2U_t + (U - 2n\xi) U_\xi + 2 [n - 1 + (\omega + 1) c] U - 4c (2c - 1) (\omega + 1)\xi = 0 \qquad (1.6)$$

3) Let us first consider self-similar solutions for $U_t = 0$, when for U we have an ordinary differential equation containing two arbitrary parameters c and n. The behavior of integral curves depends in this case on λ_1 and λ_2

$$\lambda_{1,2} = q_{1,2} - 2n, \quad q_{1,2} = 1 - (\omega + 1) c \mp [1 - (\omega + 1) c (6 - 8c) + (\omega + 1)^2 c^2]^{1/2} \qquad (1.7)$$

If λ_1 and λ_2 are different but of the same sign, we have a nodal point at the coordinate origin of plane (U, ξ) (Fig. 1, a); if the signs are different, there is a saddle point (Fig.

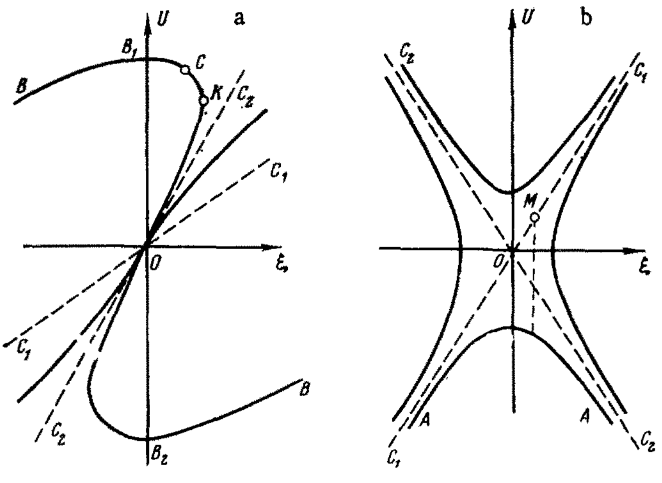

Fig. 1

1, b); if $\lambda_1 = \lambda_2 \neq 0$, we have a degenerate node, and if one of the λ_k (or both) are zeros, solutions in the plane (U, ξ) are represented by parallel straight lines. Note that curves $U = U (\xi, t)$ define velocity (pressure) distribution $u = u (x, t)$ along the axis $y = 0$. Solutions represented by straight lines passing through the singular point (shown by dash lines) are of the form

$$U = q_1\xi, \qquad U = q_2\xi \tag{1.8}$$

In the plane case $(\omega = 0)$ $q_1 = 2(1-2c)$ and $q_2 = 2c$.

In the case of a nodal point the curves are tangent to the straight line $U = q_1\xi$ when $|\lambda_1| < |\lambda_2|$ and to the line $U = q_2\xi$, when $|\lambda_1| > |\lambda_2|$. If $\lambda_1 \neq \lambda_2$, the solution of Eq. (1.6) is of the form

$$(U - q_1\xi)^{-\lambda_1}(U - q_2\xi)^{\lambda_2} = A = \text{const} \tag{1.9}$$

or in the parametric form

$$U = \frac{q_2}{q_2 - q_1}\eta + q_1 B\eta^{\varkappa}, \quad \xi = \frac{1}{q_2 - q_1}\eta + B\eta^{\varkappa}, \quad \varkappa = \frac{\lambda_1}{\lambda_2} \tag{1.10}$$

For $\lambda_1 = \lambda_2 = \lambda_0$ $(q_1 = q_2 = q)$ the solution is

$$U = \frac{q}{\lambda_0}\eta\ln\eta + \eta(1 + qB), \quad \xi = \frac{1}{\lambda_0}\eta\ln\eta + B\eta \tag{1.11}$$

In formulas (1.9) – (1.11) A and B are arbitrary constants. In the parametric form $y = y(\xi, \tau)$, $x = x(\xi, \tau)$ with $c \neq 1/2$ and $c \neq 0$ the equation of the sonic line for (1.5) is of the form

$$y^2 = \frac{m\tau^{n+1}}{2c(1-2c)}U(\xi, t) - \frac{\tau^2\lambda'(\tau)}{c(2c-1)}, \quad x = m\xi\tau^n + \frac{c}{\tau}y^2 + \lambda(\tau) \tag{1.12}$$

When $c = 0$ or $c = 1/2$ the sonic line is defined by $\xi = \xi_0(\tau)$. In what follows we consider the case of $\omega = 0$ (the analysis in the axisymmetric case is similar) and in the beginning set $\lambda'(\tau) = 0$. Then for (1.10) the equation of the sonic line assumes the form

$$y^2 = \frac{m\tau^{n+1}}{2c(1-2c)}\left[\frac{c}{3c-1}\eta + 2(1-2c)B\eta^{\varkappa}\right] = \frac{m\tau^{n+1}}{2c(1-2c)}U(\eta) \tag{1.13}$$

$$x = m\tau^n\left[\frac{1-c}{2(3c-1)(1-2c)}\eta + 2B\eta^{\varkappa}\right], \quad \varkappa = \frac{1-2c-n}{c-n}$$

We assume that $m = 1 > 0$ (for $m < 0$ the analysis is similar). The first formula of (1.13) clearly implies that it is possible to construct the sonic line for $0 < c < 1/2$ if $U > 0$, and for $c < 0$ or $c > 1/2$ if $U < 0$. The behavior of integral curves in Fig. 1 with allowance for the first formula of (1.5) shows that curves AA and C_1OC_2 in Fig. 1, b can define for $c < 0$ or $c > 1/2$ flows with LSZ in Laval nozzles. Formulas (1.13) show the variation of LSZ with time.

Taking into consideration that in the transonic approximation the equations of nozzle walls (which can be easily presented in a parametric form) are of the form

$$y = y_0 + \varepsilon f(x, \tau), \quad \frac{\partial f}{\partial x} = v(y_0, x, \tau), \quad y_0 = \text{const}, \quad \varepsilon \ll 1 \tag{1.14}$$

we conclude on the basis of (1.13) that for $n > -1$ the LSZ which at the initial instant occupy a part of the nozzle throat vanish after a certain time, and the stream becomes everywhere subsonic, i.e. solutions for $n > -1$ define flows with vanishing LSZ. An example of such flow (for $n = 2$, $c = -3$ and $B > 0$) is presented qualitatively in Fig. 2, a. On the other hand, when $n < -1$, the development of LSZ is observed (an example of this is shown in Fig. 2, b for $n = -3/2$, $c = 4$ and $B < 0$). It is evident from the second formula of (1.13) that with increasing time the LSZ widens for $n > 0$ and narrows for $n < 0$.

Note that it is possible to substitute in all formulas $(\tau + \tau_0)$ for τ, where $0 \leqslant \tau < \infty$ and $\tau_0 > 0$. According to (1.13) flows with LSZ which merge at the nozzle axis for $\tau \to \infty$ and $n < -1$ take place for $\lambda'(\tau) = 0$. When $\lambda'(\tau) \neq 0$, then for the same values of c ($c < 0$ or $c > {}^1/_2$) the LSZ merge for $\tau = \tau_1$ at the nozzle axis and the supersonic zone subsequently occupies a whole segment of the axis (or the inverse process takes place with increasing time). Such LSZ are readily constructed by formulas (1.12) and (1.13). Note that solutions defined by curves with closed sections $U > 0$

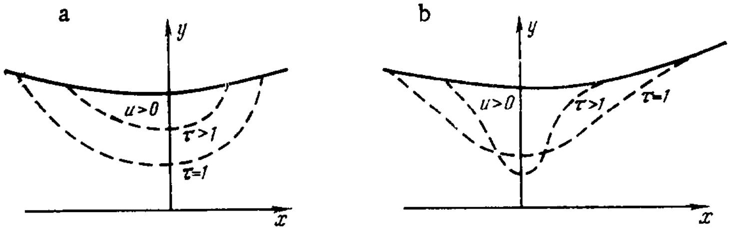

Fig. 2

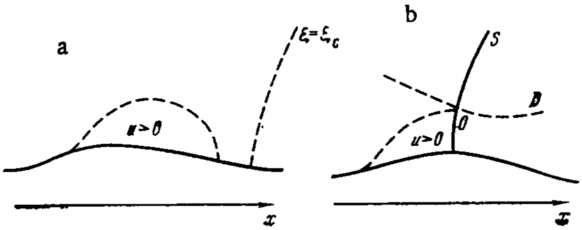

Fig. 3

along the axis $y = 0$ may describe flows with LSZ close to the profile (Fig. 3, a). Examples of solutions which have such sections are given in Fig. 1, a by curves BOB and, if $\lambda'(\tau) > 0$, by curves AA and C_1OC_2 in Fig. 1, b. In this case it follows from (1.13) that for such flows $0 < c < {}^1/_2$. However for such c the solution for flows of this kind is defective (contains regions of ambiguity and nonexistence of solutions). The pattern of flow in proximity of the profile for $c = {}^1/_4$, $n = {}^5/_8$, $B > 0$ and $y_0 = 3.35$ is shown qualitatively in Figs. 3, a. The integral curve BB_1C in Fig. 1, a ($\xi_C \leqslant \xi_K$, $U'(\xi_K) = \infty$) corresponds to this flow. It is not possible to construct the flow beyond $\xi = \xi_C$ using a solution of the class of (1.5). It is apparently possible to extend that solution beyond $\xi = \xi_C$ by using a solution more general than (1.5) and (1.9) and admitted by system (1.1), which is of the form

$$u_* = U_0(\xi) + U_2(\xi)\, y_*^2, \quad x_* = X_0(\xi) + X_2(\xi)\, y_*^2 \qquad (1.15)$$
$$v_* = V_1(\xi)\, y_* + V_3(\xi)\, y_*^3$$

4) Let us consider flows with LSZ and compression shocks. Conditions at the shock front are of the form

$$2\frac{\partial x}{\partial \tau} + \left(\frac{\partial x}{\partial y}\right)^2 = \frac{1}{2}\left(u^{(1)} + u^{(2)}\right), \quad v^{(1)} + u^{(1)}\frac{\partial x}{\partial y} = v^{(2)} + u^{(2)}\frac{\partial x}{\partial y} \qquad (1.16)$$

where the superscripts relate to flows on opposite sides of the shock. Let us assume that the solution at one side of the shock is of the form (1.5) and at the other of the form (1.15) (the case of $U_t = 0$ is considered). Then, satisfying conditions (1.16) at the compression shock $\xi = \xi_0 = \text{const}$ and $x = \xi_0\tau^n + (c / \tau)y^2 + \lambda(\tau)$, we obtain $(m = 1)$

$$U_0(\xi_0) + U(\xi_0) = 4n\xi_0, \quad U_2(\xi_0) = 2c(2c-1), \quad X_0(\xi_0) = \xi_0 \tag{1.17}$$
$$X_2(\xi_0) = c$$
$$V_1(\xi_0) = 4c(2c-1)\xi_0 - 2cU_0(\xi_0), \quad V_3(\xi_0) = {}^8/_3 c(c-1)(2c-1)$$

Conditions at the shock front $\xi = \xi_0(\tau)$ in the case when coefficients in (1.5) and (1.15) also depend on τ are analogous. The shock intensity is defined according to (1.17) by the expression

$$u^{(1)} - u^{(2)} = 2U(\xi_0) - 4n\xi_0 \tag{1.18}$$

If the solution upstream of the shock wave is taken in the form (1.5) and (1.10), according to (1.18) it is necessary to supplement (1.17) by the following condition:

$$U(\xi_0) > 2n\xi_0 \tag{1.19}$$

which shows that the shock is a compression one. If the solution (1.5) and (1.10) is taken downstream of the shock, the inequality sign must be reversed.

Analysis of condition (1.17) and of the behavior of integral curves shows that it is possible to construct a great number of flows of various kinds with compression shocks. Let us examine some of these. The flow with LSZ in the proximity of the profile, which ends in a compression shock ($c = {}^1/_4$, $n = {}^5/_8$ and $B > 0$) is shown in Fig. 3, b. Upstream of the shock $\xi = 0$ the solution is represented by curve BB_1 and downstream of it by curve B_2B (Fig. 1, a). Thus the three-valued region is replaced by a compression shock. However, in accordance with (1.18), the intensity of the compression shock is constant, does not decrease with increasing y, and cannot be extended into region D which lies above point O of intersection of the sonic line with the shock front. From the physical point of view the considered class of solutions is evidently unsuitable for defining flows near a profile.

Flows with LSZ bounded by a compression shock $\xi = \xi_0$ are readily constructed for $\lambda_1 = 0$ or $\lambda_2 = 0$. One of such possible flows is of the form shown in Fig. 3, b. Solutions $U = q_k\xi$ (represented by straight lines C_kC_k $(k = 1, 2)$) can be successfully used for constructing flows with shock waves. If, for example, we take upstream of the shock $\xi = \xi_0 > 0$ one of these solutions and downstream of it the other, we obtain a flow with LSZ in a Laval nozzle ending in a shock wave. Satisfying the first of conditions (1.17) (the remaining are automatically satisfied), we obtain $c = 1 - 2n$. In addition, by satisfying condition (1.19), specifying that q_1 and q_2 must be of different signs (i.e. that the indicated straight lines must lie in different quarters), and selecting $\xi_0 > 0$, we come to the conclusion that, if upstream of the shock we take the solution $U = q_1\xi$, then $c < 0$, while with the solution $U = q_2\xi$ we have $c > {}^1/_2$. If ξ_0 is fairly small, the flow downstream of the shock can be supersonic, which with increasing distance from the shock becomes subsonic. If ξ_0 is reasonably great, the flow is subsonic immediately downstream of the shock. When $\xi_0 = 0$ the shock vanishes and the curve $\xi_0 = 0$ is then a characteristic along which the two solutions $U = q_1\xi$ and $U = q_2\xi$ merge. In Fig. 1, b such solution is represented by the broken line C_1OC_2. It represents a shock-free flow in a nozzle with the LSZ merging at one point of the nozzle axis. Flows in which supersonic zones downstream of the shock lie close to the walls without reaching the nozzle axis at any point, while in the proximity of the axis the flow downstream of the shock is subsonic. Such nozzles were investigated in [6] in the stationary case. Such flow can be readily constructed with the use of one of the solutions $U = q_k \xi$, upstream of the shock and the general solution (1.9) downstream of it.

5) Let us now consider solution (1.5) for the general case of $U = U(\xi, t)$. Using (1.10) it is easy to obtain the first two integrals of Eq. (1.6), which yields for $\lambda_1 \neq \lambda_2$ the general solution

$$F\,[(U - q_1\xi)^{-\lambda_1}(U - q_2\xi)^{\lambda_2},\ (U - q_1\xi)\exp(-{}^1/_2\lambda_2 t)] = 0 \qquad (1.20)$$

where F is an arbitrary function of two arguments. Representing solution (1.20) as resolved for the second argument, we obtain it in the form $t = t(U, \xi)$. Note that here Eq. (1.6) is linear with respect to function $t(U, \xi)$. In the parametric form convenient for computations solution (1.20) is written in the form (1.10) in which B is an arbitrary function of $\eta \exp(-{}^1/_2\lambda_2 t)$.

If $\lambda_1 = \lambda_2 = \lambda_0$ $(q_1 = q_2 = q)$, the general solution of Eq. (1.6) is of the form

$$F\left[\ln(U - q\xi) - \frac{\lambda_0\xi}{U - q\xi},\ (U - q\xi)\exp(-{}^1/_2\lambda_0 t)\right] = 0 \qquad (1.21)$$

In parametric form the solution is provided by formulas (1.11), where $B(\eta \exp(-\lambda_0 t/2))$ is an arbitrary function.

2. Let us consider one more class of solutions for the system of Eqs. (1.1). We pass in (1.1) to the new variables

$$u = e^{2n\tau}u_*(x_*, y_*, \tau) + 2\lambda'(\tau), \qquad v = e^{3n\tau}v_*(x_*, y_*, \tau) + \frac{4}{\omega+1}\lambda''(\tau)\,y \qquad (2.1)$$

$$x_* = [x - \lambda(\tau)]e^{-2n\tau}, \qquad y_* = ye^{-n\tau}$$

Solutions which define flows with LSZ also exist for (2.1). Written in physical variables x, y, τ they are of the form

$$u = me^{2n\tau}U(\xi, \tau) + 4c^2y^2 + 2\lambda'(\tau), \qquad x = m\xi e^{2n\tau} + cy^2 + \lambda(\tau) \qquad (2.2)$$

$$v = 2cme^{2n\tau}(4c\xi - U)\,y + \frac{16}{\omega+3}c^3y^3 + \frac{4}{\omega+1}\lambda''(\tau)\,y$$

Function U satisfies equation

$$2U_\tau + (U - 4n\xi)U_\xi + 2\,[2n + (\omega + 1)\,c]\,U - 8(\omega + 1)c^2\xi = 0 \qquad (2.3)$$

Let us first consider the self-similar case of $U = U(\xi)$ whose solution for $\lambda_1 \neq \lambda_2$ is of the form (1.9), (1.10), where

$$\lambda_k = q_k - 4n, \quad q_{1,2} = (-\omega - 1 \pm \sqrt{\omega^2 + 10\omega + 9})c \qquad (2.4)$$

The solution for $\lambda_1 = \lambda_2$ $(c = 0)$ is also readily derived. Investigation of the behavior of integral curves (and of related kinds of flows) in terms of λ_1 and λ_2 is exactly as in the case of (1.9) (the reasoning is repeated literally). However, unlike in the case of (1.5), (1.6) the behavior of curves does not depend on c and n but on the ratio $\alpha = n/c$. For $\alpha < -1$ or $\alpha > {}^1/_2$ we have a node at point $U = \xi = 0$ and for

$-1 < \alpha < {}^1/_2$ a saddle. The asymptotes $U = q_k\xi$ always lie in different quarters. The equation of the sonic line is of the form (we assume $m = 1$)

$$y^2 = -\frac{\lambda'(\tau)}{2c^2} - \frac{1}{4c^2} e^{2n\tau} U(\eta), \quad x = -\frac{\lambda'(\tau)}{2c} + e^{2n\tau}\left[\xi(\eta) - \frac{1}{4c} U(\eta)\right] \quad (2.5)$$

In [4] sonic lines (2.5) were constructed for $n = 0$. Formulas (2.5) provide a clear picture of the change of sonic lines with time. They show that for $n > 0$ (and related function $\lambda(\tau)$) solutions define the process of LSZ "attenuation" in Laval nozzles, and for $n < 0$ that of their development.

As in the case of (1.5) it is possible to construct with the use of solution (2.2) the flow with compression shocks in Laval nozzles. For this the equation of the shock is specified in the form

$$\xi = \xi_0, \quad x = \xi_0 e^{2n\tau} + cy^2 + \lambda(\tau) \quad (2.6)$$

In this case the shock wave shape does not change with time, while for solutions (1.5) its shape defined by $x = \xi_0\tau^n + (c / \tau)y^2 + \lambda(\tau))$ varies with time. Assuming that at one side of the shock the solution is of the form (1.15), (2.1) and at the other of the form (2.2), we readily obtain conditions at the shock front, similarly to (1.17). Let us consider the particular case when upstream of the shock the solution is of the form (2.2) with $U = U_1(\xi)$ and downstream of it with $U = U_2(\xi)$. From condition (1.16) at the shock front we then obtain the single requirement

$$U_1(\xi_0) + U_2(\xi_0) = 8n\xi_0 \quad (2.7)$$

Stipulation for the shock to be a compression one yields the condition

$$U_1(\xi_0) > 4n\xi_0$$

Note that for (2.2) the intensity of shock waves is independent of y but varies with time. In this class of solutions the construction of the flow with a compression shock in a nozzle in accordance with (2.7) is elementary. Such flows may, for instance, be represented by curve C_1OMA (Fig. 1, b). It is interesting that in this case the flow upstream of the compression shock is stable (for $\lambda = 0$), while downstream of it it changes with time. The change in the stream can be readily observed with the use of formulas (2.5) and (2.6). We point out that solution (2.2) is a generalization of solutions considered in [1, 2, 4] which are obtained from (2.2) by setting $n = 0$. As already noted above, solutions (2.2) can be extended to the three-dimensional case

$$u = e^{2n\tau}U(\xi, \tau) + a_1y^2 + a_2z^2 + 2\lambda'(\tau), \quad x = \xi e^{2n\tau} + c_1y^2 + c_2z^2 + \lambda(\tau) \quad (2.8)$$

Thus solution (2.8) yields as a particular case for $n = 0$ all known solutions of this kind [1 – 4], for example, solutions for steady flows. For $n = 0$ the conditions at the shock front and along characteristics represent conditions for steady flows. Note that the more general class of solutions (2.1) has the same property when $\partial u_* / \partial\tau = \partial v_* / \partial\tau = \lambda = 0$. In the general case of (2.2) when $U = U(\xi, \tau)$ by determining the first integrals for (2.3), we obtain the general solution for $c \neq 0$ $(\lambda_1 \neq \lambda_2)$ in the form (1.20), (2.4) in which τ is substituted for t. The general solution for $\lambda_1 = \lambda_2$ $(c = 0)$ is readily obtained in a form similar to (1.21).

3. We conclude with the following remark. Let us consider the system of equations

$$Au_\tau + G^{(1)}u_x + F^{(1)}v_x + F^{(2)}u_y + G^{(2)} = 0, \quad F^{(3)}u_x + Bv_x + Cu_y + F^{(4)} = 0 \quad (3.1)$$

$$G^{(k)} = \sum_{n=0}^{2} G_n^{(k)} y^n + G_4^{(k)} x + G_5^{(k)} u, \qquad F^{(k)} = F_1^{(k)} + F_2^{(k)} y$$

where the coefficients $A, B, C, G_n^{(k)}$ and $F_n^{(k)}$ are generally some functions of τ. System (3.1) includes the following particular cases (plane and axisymmetric flows are considered) : (a) equations (1.1), (b) equations for transonic vortex flows [7], (c) equations for transonic flows of chemically active gas [8], (d) equations for short waves [9], and (e) equations of magnetohydrodynamics for transonic and hypercritical flows [10].

The system of Eqs. (3.1) has a two-parameter class of solutions

$$u = \sum_{k=0}^{2} U_k(\xi, \tau)\eta^k, \quad v = \sum_{k=0}^{3} V_k(\xi, \tau)\eta^k \tag{3.2}$$

$$x = \sum_{k=0}^{2} X_k(\xi, \tau)\eta^k, \quad y = Y_0(\xi, \tau) + Y_1(\xi, \tau)\eta$$

For $Y_0 = 0$ $(\eta = y)$ we obtain from (3.2) one-parameter solutions which include, as a particular case, solutions of the form (1.3). For $Y_1 = 0$ $(\xi = y)$ in (3.2) contains solutions

$$u = \sum_{k=0}^{2} U_k(y, \tau) x^k, \qquad v = \sum_{k=0}^{3} V_k(y, \tau) x^k \tag{3.3}$$

As an example, we present some particular solutions of equations [8]

$$uu_x - v_y - \frac{\omega}{y} v + \alpha u = 0, \quad u_y = v_x, \quad \alpha = \text{const} \tag{3.4}$$

An example of solutions of the form (3.2) for $Y_0 = 0$ of system (3.4) is the solution

$$u = mU(\xi) + 4c^2y^2, \quad x = m\xi + cy^2 \tag{3.5}$$

$$v = 2cm(4c\xi - U)y + \frac{4c^2}{\omega+3}(4c+1)y^3$$

which defines a flow with LSZ in Laval nozzles.

Function U is specified by formulas (1.9) and (1.10), where

$$\frac{\lambda_{1,2}}{c} = \frac{q_{1,2}}{c} = -\left(\frac{\alpha}{2c} + \omega + 1\right) \pm \sqrt{\left(\frac{\alpha}{2c} + \omega + 1\right)^2 + 8(\omega + 1)}, \quad q_1 \neq q_2 \tag{3.6}$$

The solution for $q_1 = q_2$ is readily derived. For $\alpha = 0$ we obtain a solution of system (1.1) for a chemically active gas. The behavior of integral curves is qualitatively represented in Fig. 1, b. The solution of system (3.4) which in the case of a free sonic stream of gas flowing past a profile defines the flow in the neighborhood of the intersection point of two sonic lines (one of which for $\omega = 0$ is specified by $y = 0$ and for $\omega = 1$ by $r = r_0 \neq 0$) is an example of solutions of the form (3.3). For simplicity we set in (3.4) $\alpha = 1$ and obtain for u the formula

$$u = y(2ax + {}^1/_3 a^2y^3 + {}^a/_3 y^2 + c), \qquad \omega = 0 \tag{3.7}$$

$$u = \xi[{}^2/_3 x + c + {}^1/_9 r^2(\xi - {}^1/_2)], \quad \xi = \ln r + {}^3/_2 b, \quad \omega = 1$$

In formulas (3.5) – (3.7) a, b, c and m are arbitrary constants. Solutions of the form (3.5) and (3.7) can be readily derived also for equations which define vortex flows [7].

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ON THE HODOGRAPH METHOD FOR AXISYMMETRIC TRANSONIC FLOWS OF GAS

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Behavior of a transonic stream of gas perturbed by a body of revolution is investigated at some distance from that body in the hodograph plane. An asymptotic expansion of the Legendre potential is derived.

The flow of a perfect gas stream, whose velocity at infinity is constant and close to the speed of sound, past a slender body of revolution is considered. The problem of attenuation of perturbations induced by the body of revolution in the transonic stream in the region upstream of compression shocks at some distance from the body is analyzed.

An asymptotic expansion of the velocity potential in the considered region was obtained in [1] in variables of the physical plane of flow. However hodograph variables proved to be more convenient in a number of problems, since the equation of shock wave in these variables becomes determinate. Because